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Relativistic statistical arbitrage

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Recent advances in high-frequency financial trading have made light propagation delays between geographically separated exchanges relevant. Here we show that there exist optimal locations from which to coordinate the statistical arbitrage of pairs of spacelike separated securities, and calculate a representative map of such locations on Earth. Furthermore, trading local securities along chains of such intermediate locations results in a novel econophysical effect, in which the relativistic propagation of tradable information is effectively slowed or stopped by arbitrage.

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I. INTRODUCTION

Recent advances in high-frequency financial trading have brought typical trading latencies below 500 μs [1], at which point light propagation delays due to geographically separated information sources become relevant for trading strategies and coordination (e.g., it takes 67 ms, over 100 times longer, for light to travel between antipodal points along the Earth's surface). Moreover, as trading times continue to decrease in coming years (e.g., latencies in the microseconds are already being targeted by traders [2]), this feature will become even more pronounced.

Here we report a relativistic generalization of statistical arbitrage trading strategies [3–5] for spacelike separated trading locations. In particular, we report two major findings. First, we prove that there exist optimal intermediate locations between trading centers that host cointegrated [6,7] securities such that coordination of arbitrage trading from those intermediate points maximizes profit potential in a locally auditable manner. For concreteness, we calculate a representative map of such intermediate locations assuming simplified behavior by securities at the world's largest existing trading centers. Second, we show that if such intermediate coordination nodes are themselves promoted to trading centers that can utilize local information, a novel econophysical effect arises wherein the propagation of security pricing information through a chain of such nodes is effectively slowed or stopped.

Financial background

Cointegration provides a particularly useful notion of correlation between time series. A pair of time series x, y is said to be *cointegrated* if neither x nor y is a stationary stochastic process, but some nontrivial linear combination of x and y (given by a *cointegrating vector*) is stationary [6,7]. Because of this stationarity, the linear combination described by the cointegrating vector will exhibit long-term reversion toward an equilibrium value [6–9].

Within financial markets, the relevant time series are typically the logarithms of the prices (log-prices) of financial instruments. Some of the simpler instances of cointegrated time series arise from interchangeable financial products, whose prices will therefore not drift far apart [6]. Such pairs should be expected for commodities or foreign currencies that are traded in multiple markets, and for stocks that are cross-listed (e.g., via American or Global Depository Receipts) [6,8,10]. Likewise, futures and spot prices form a cointegrated pair for stock indexes [11] and foreign currencies [12]. More generally, cointegrated time series have been found among pairs of highly correlated stocks [9,13], larger portfolios of stocks [14], and groups of foreign currency exchange rates [15]. While entire financial markets are not thought to themselves exhibit mean reversion in log-prices [16,17], rapid convergence to equilibrium has been found to hold empirically for the difference in log-prices of cross-listed securities (up to a small premium) [10,18–20], and to a lesser, but substantial, degree for linear combinations of foreign currency exchange rates [15,21] and highly-correlated stock pairs [13,22].

To describe the behavior of correlated financial instruments, we will use the Vasicek model [23], which is based on elastic Brownian motion given by an Ornstein-Uhlenbeck process [24] and which can be viewed [25] as a continuous limit of the Ehrenfest urn model of diffusion [26]. The Vasicek model was originally introduced to model a tendency toward long-term mean reversion in the term structure of interest rates, but has proven to be a flexible general model for financial assets that exhibit mean reversion [27–29]. Ornstein-Uhlenbeck processes, and their refinements, have also been used to model stochastic (rather than constant) volatility within a wide variety of more complex models for interest rates and stock prices [30–40]. More recently, the Vasicek model has been used to model pairs trading strategies [41–43] (e.g., between stocks for highly correlated companies in the same industry).

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II. DISCRETE MODEL

We begin by considering the coordination of trades involving two correlated, but spacelike separated, financial instruments from a single intermediate location. For concreteness, we will speak of securities (e.g., stocks or bonds), although the same analysis applies to any rapidly traded and highly liquid financial instruments, including derivatives (e.g., options, futures, forwards, or swaps) on an underlying asset (e.g., stock, currency, commodity, or index).

A. Spatially-separated Vasicek model

Under the Vasicek model for pairs trading, one typically considers two colocated securities whose log-prices are cointegrated [13,41]. Suppose the cointegrating linear combination $r(t)$ has long-term mean b, speed of reversion a, and instantaneous volatility σ. Then $r(t)$ is given by

$$dr(t) = a(b - r(t))dt + \sigma dW(t), \tag{1}$$

where $W(t)$ is a Wiener process determined by local conditions.

Here we instead consider the case of two trading centers, with a spacelike geodesic separation of $c\tau$, that respectively host cointegrated securities having log-prices $x(t), y(t)$, local speeds of reversion a_x, a_y, and instantaneous volatilities σ_x, σ_y. In the case of cross-listed and dual-listed securities (which are already spacelike separated, hence amenable to this analysis), one can take the cointegrating linear combination to be simply the difference in log-prices (i.e., cointegrating vector $(1,-1)$), which will have long-term mean $b=0$ because the securities are essentially interchangeable [6,18,20].

Even in more general pairs trading, the cointegrating vector can often be taken to be approximately $(1,-1)$, with drift term $b \approx 0$ [13]. Beyond pairs, one could construct such cointegrated linear combinations of securities—one at each spatially separated site (e.g., using principal component analysis [44] at each site). Furthermore, many other tradable cointegrated time series also exhibit vector $(1,-1)$ and drift 0 (e.g., in futures and spot prices for stock indexes [11] and foreign currencies [12], and in macroeconomic indicators [45,46]). Hence we will work under the assumption that $x-y$ is stationary with long-term mean 0.

Let $V(t), W(t)$ be independent Wiener noise sources. Then the decoupled Vasicek processes are

$$dx(t) = -a_x x(t)dt + \sigma_x dV(t), \tag{2a}$$

$$dy(t) = -a_y y(t)dt + \sigma_y dW(t). \tag{2b}$$

Now consider an intermediate node along the geodesic at distance $c\Delta t$ from the center hosting the security with log-price x, capable of issuing pair trading orders instantaneously at communication speed c based on locally available information (see Fig. 1).

Traditionally, such pair trades are triggered when the ratio between the two security prices leaves its historical statistical bound [41,42]. The securities are then unloaded after they equilibrate, and so ultimate profit derives from maximizing the statistical significance of the ratio at the times of execution. In contrast, in our arrangement, profit is also determined by the locations of execution and we will therefore want to identify intermediate trading node locations (Δt) that maximize the security ratio at the respective executions.

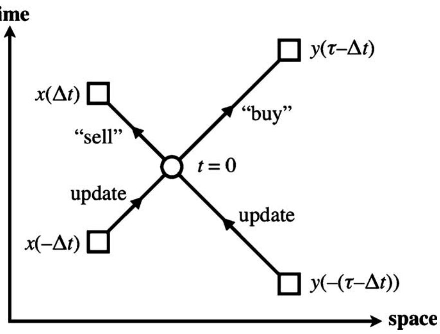

FIG. 1. Space-time diagram of relativistic arbitrage transaction. Security price updates from spacelike separated trading centers (squares) arrive with different light propagation delays at an intermediate node (circle) at time $t=0$, whereupon the node may issue a pair of "buy" and "sell" orders back to the centers.

Of course, any such intermediate node would lie on the past light cones of both trading centers, which could together emulate any prearranged coordination strategy by the node. However, emulation would be problematic: although the net position would remain nearly market neutral, this fact could not be guaranteed at either center in real time. A trader emulating an intermediate strategy would occasionally hold a very long position at one center and very short at the other, and so it is essential that the trader be able to convince the financial exchanges at both centers that the net position is small in comparison with the position at either center to avoid onerous capital requirements.

In traditional trading with a single exchange, positions that offset each other typically incur reduced margin requirements [47,48]. However, a dishonest trader in the relativistic scenario, claiming falsely to implement an emulated strategy, could make unfair use of these reduced requirements, if allowed to do so, and hence only a guarantee of both transactions should suffice. With an actual intermediate node, this guarantee could be provided by a local audit. For example, immediately following a pair trade, the intermediate node could transmit a message to each center, cryptographically signed by a trusted local agency, certifying that an offsetting order to the other center (that is guaranteed to be filled, such as a market order) had just been issued. In contrast, a trader not using an intermediate node could not issue such audits in real time. In order to avoid capital requirements commensurate with the possible position accumulated during the light propagation delay, such a trader could prearrange audits (e.g., by making the entire strategy public in advance). In doing so, though, that trader would then become unable to adapt strategies based on local events, thereby incurring additional risk compared to a local trader at an intermediate node. Therefore, there is substantial justification for arbitrage from true intermediate nodes.

B. Calculation of optimal intermediate trading locations

Returning to our analysis of the optimal intermediate node location (Δt), let us now assume, without loss of generality, that $x(-\Delta t) > y(-(\tau-\Delta t))$, so that we are trying to maximize the security ratio $e^{R(\Delta t)} \equiv e^{x(\Delta t)}/e^{y(\tau-\Delta t)}$ at the respective local times of execution in order to ensure the sign fidelity of the trade. Now, from the perspective of the intermediate node at $t=0$, the expectations of x,y will decay exponentially [23] back to their long-term (zero) means from their instantaneously known values, $x(-\Delta t), y(-(\tau-\Delta t))$:

$$\langle R(\Delta t)\rangle = x(-\Delta t)e^{-2a_x\Delta t} - y(-(\tau-\Delta t))e^{-2a_y(\tau-\Delta t)}. \quad (3)$$

The expected security ratio will reach an extremum when Δt satisfies

$$0 = \frac{d\langle R(\Delta t)\rangle}{d\Delta t} = -e^{-2a_x\Delta t}(\dot{X} + 2a_x X) - e^{-2a_y(\tau-\Delta t)}(\dot{Y} + 2a_y Y),$$

where $X \equiv x(-\Delta t)$, $\dot{X} \equiv \dot{x}(-\Delta t)$, $Y \equiv y(-(\tau-\Delta t))$, and $\dot{Y} \equiv \dot{y}(-(\tau-\Delta t))$. We obtain a degenerate solution class when reversion occurs at a particular speed in the absence of noise, given by

$$\dot{X} + 2a_x X = 0 = \dot{Y} + 2a_y Y, \quad (4)$$

and a physically relevant solution class, given by

$$\frac{\Delta t}{\tau} = \frac{a_y}{a_x + a_y} + \frac{1}{2(a_x + a_y)\tau}\ln\left[-\frac{\dot{X} + 2a_x X}{\dot{Y} + 2a_y Y}\right]. \quad (5)$$

We note that the first term in Eq. (5) depends only on the long-term behavior of the securities, while the second term is sensitive to the instantaneous behavior of the securities.

Having identified the extrema, we now constrain our search to maxima using the inequality

$$0 > \frac{d^2\langle R(\Delta t)\rangle}{d\Delta t^2} = e^{-2a_x\Delta t}[2a_x(\dot{X} + 2a_x X) + \ddot{X} + 2a_x\dot{X}] - e^{-2a_y(\tau-\Delta t)}[2a_y(\dot{Y} + 2a_y Y) + \ddot{Y} + 2a_y\dot{Y}],$$

where $\ddot{X} \equiv \ddot{x}(-\Delta t)$ and $\ddot{Y} \equiv \ddot{y}(-(\tau-\Delta t))$. It follows that, for there to be a global maximum at Δt_{max} where Δt_{max} solves Eq. (5) and $0 \le \Delta t_{max} \le \tau$, it is sufficient (although not necessary) that the following condition be satisfied over the same range ($0 \le \Delta t \le \tau$):

$$4a_x^2 X + 4a_x\dot{X} + \ddot{X} < 0 < 4a_y^2 Y + 4a_y\dot{Y} + \ddot{Y}. \quad (6)$$

Heuristically, these conditions are satisfied by sufficiently sharp price fluctuations including, for example, the following generic class of locally quadratic price fluctuations:

$$x(t) = k_0 - k_1(\Delta t + \tau)^2, \quad (7a)$$

$$y(t) = -k_2 + k_3(\Delta t + \tau)^2, \quad (7b)$$

where

$$2a_x^2 k_0 < k_1 < k_0/\tau^2,$$

$$2a_y^2 k_2 < k_3 < k_2/\tau^2,$$

$$a_x, a_y < 1/(\tau\sqrt{2}),$$

which correspond to fluctuations whose characteristic frequency f lies in the broad range

$$\frac{\max(a_x, a_y)}{\pi} < f < \frac{\tau^{-1}}{\pi\sqrt{2}}. \quad (8)$$

Such fluctuations should be common in the high-frequency trading of securities, as we now calculate. For maximally distant points on Earth, $\tau \approx 67$ ms, and for adjacent trading centers, τ is even smaller (e.g., $\tau \approx 1.1$ ms for London and Paris). These imply lower bounds (within this example solution class) on the characteristic time between exploitable fluctuations, f^{-1}, of approximately 300 ms for distant trading centers, 5 ms for adjacent trading centers, and much lower still for multiple markets within a single city (with different colocation facilities). On the other hand, typical mean-reversion times (which determine approximate values of a_x^{-1}, a_y^{-1}) are typically far longer (e.g., a half-life of 1–5 days for the equilibration of cross-listed shares [18,20]). Even in situations where equilibrium is reached in minutes or hours, Eq. (8) therefore determines a wide interval of relevant frequencies. Crucially, this range includes the typical time scales exploited in high-frequency trading (e.g., potentially profitable fluctuations in the 10 ms–10 s range [50]). These calculations suggest that while arbitrage between distant sites at the smallest relevant time scales has been technically possible for several years, the fastest relevant arbitrage between nearby sites has only recently become technologically feasible.

C. Example: Optimal trading locations on Earth

For concreteness, we now calculate an example set of optimal locations for intermediate trading nodes under the simplifying assumption that such fluctuations are high frequency [$f \gg \max(a_x, a_y)/\pi$, implying that $|\frac{\dot{X}}{X}| \sim 2\pi f \gg 2\max(a_x, a_y)$] and comparable in magnitude but oppositely signed ($X \sim -Y$). Under these assumptions, the behavior-dependent logarithmic term in Eq. (5) vanishes, and the optimal intermediate location simplifies to an average of the two center locations weighted by speeds of reversion,

$$\Delta t = \tau a_y/(a_x + a_y). \quad (9)$$

Let us furthermore assume that, based on dimensional reasoning, the speeds of reversion scale with market turnover velocities [49].

Under these assumptions, the optimal intermediate locations are therefore midpoints weighted by turnover velocity. In Fig. 2 we compute the optimal intermediate locations, as such weighted midpoints [Eq. (9)], for all pairs of 52 major exchanges based on 2008 turnover velocity data reported by the World Federation of Exchanges [49]. In practice, intermediate locations could be calculated for specific pair trades using more precisely estimated reversion speeds, or for more complicated transactions involving more than two securities.

Note that while some nodes are in regions with dense fiber-optic networks, many others are in the ocean or other sparsely connected regions, perhaps ultimately motivating the deployment of low-latency trading infrastructure at such remote but well-positioned locations.

III. CONTINUUM MODEL

In the previous example, the simplifying assumption of rapid fluctuations allowed us to ignore the instantaneous behavior of individual securities. When instantaneous behavior is taken into account, all points on the connecting geodesics between trading centers become potentially profitable locations for arbitrage nodes. Moreover, once multiple competing nodes accumulate near a given intermediate location, there is the opportunity to execute trades locally among themselves, effectively creating a new trading center at that location. Nodes at such intermediate centers would be able to execute local trades and act on local information or events (e.g., weather) immediately. Consequently, we now extend our analysis of relativistic statistical arbitrage to chains of trading centers along geodesics.

A. Chain of trading centers

Specifically, let us consider a one-dimensional chain of trading centers with spacing h and communication speed c that host correlated local securities whose log-prices sample some continuous function $u(x,t)$ of geodesic location x and time t. Furthermore, assume that the chain is sufficiently dense that $u(x,t)$ varies slowly on length scales of h and time scales of h/c. For simplicity in characterizing the propagation of pricing information through the nodes, let us also assume that the local Wiener noise sources are currently switched off and that, due to local arbitrage, the security log-prices revert with common speed a toward the instantaneously observed average of log-prices at nearest neighbor centers. We begin our analysis with three consecutive trading centers at locations $x-h$, x, $x+h$. Under these assumptions, the Vasicek process for reversion to the instantaneously observed (i.e., retarded by time h/c) mean of nearest neighbor log-prices is given by

$$du(x,t) = a\left[\frac{u(x+h,t-h/c)+u(x-h,t-h/c)}{2} - u(x,t)\right]dt.$$

Utilizing the assumption of slow variation on length scales of h and time scales of h/c to expand u to second-derivative order, we find

$$\begin{aligned}
\frac{\partial u(x,t)}{\partial t} &= \frac{a}{2}\left[u\left(x+h,t-\frac{h}{c}\right) - u(x,t) + u\left(x-h,t-\frac{h}{c}\right) - u(x,t)\right] \\
&\approx \frac{a}{2}\left[h\frac{\partial u\left(x+\frac{h}{2},t-\frac{h}{2c}\right)}{\partial x} - \frac{h}{c}\frac{\partial u\left(x+\frac{h}{2},t-\frac{h}{2c}\right)}{\partial t} - h\frac{\partial u\left(x-\frac{h}{2},t-\frac{h}{2c}\right)}{\partial x} - \frac{h}{c}\frac{\partial u\left(x-\frac{h}{2},t-\frac{h}{2c}\right)}{\partial t}\right] \\
&= \frac{a}{2}\left[h\frac{\partial u\left(x+\frac{h}{2},t-\frac{h}{2c}\right)}{\partial x} - h\frac{\partial u\left(x-\frac{h}{2},t-\frac{h}{2c}\right)}{\partial x} - \frac{h}{c}\frac{\partial u\left(x+\frac{h}{2},t-\frac{h}{2c}\right)}{\partial t} - \frac{h}{c}\frac{\partial u\left(x-\frac{h}{2},t-\frac{h}{2c}\right)}{\partial t}\right] \\
&\approx \frac{a}{2}\left[h^2\frac{\partial u\left(x,t-\frac{h}{2c}\right)}{\partial x^2} - \frac{2h}{c}\frac{\partial u\left(x,t-\frac{h}{2c}\right)}{\partial t} + O\left(\frac{\partial^3 u}{\partial x^2\,\partial t}\right)\right] \approx a\left[\frac{h^2}{2}\frac{\partial^2 u\left(x,t-\frac{h}{2c}\right)}{\partial x^2} - \frac{h}{c}\frac{\partial u\left(x,t-\frac{h}{2c}\right)}{\partial t}\right]
\end{aligned}$$

Shifting time forward by $h/2c$, we obtain

$$\frac{\partial u\left(x,t+\frac{h}{2c}\right)}{\partial t} \approx a\left[\frac{h^2}{2}\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{h}{c}\frac{\partial u(x,t)}{\partial t}\right],$$

and so, expanding the left hand side, we recover a form,

$$\frac{\partial u(x,t)}{\partial t} + \frac{h}{2c}\frac{\partial^2 u(x,t)}{\partial t^2} \approx \frac{ah^2}{2}\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{ah}{c}\frac{\partial u(x,t)}{\partial t}, \tag{10}$$

that can be expressed as a homogeneous telegraph equation [51–57],

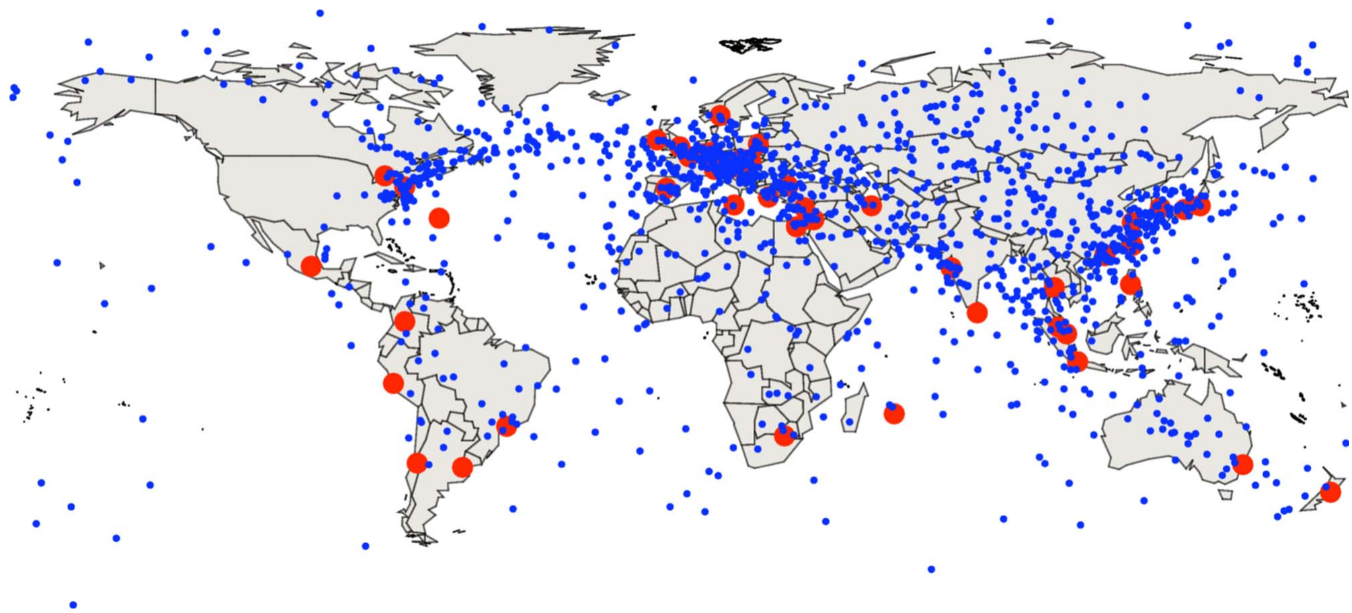

FIG. 2. (Color online) Optimal intermediate trading node locations (small circles) for all pairs of 52 major securities exchanges (large circles), calculated using Eq. (9) as midpoints weighted by turnover velocity (from 2008 data reported by the World Federation of Exchanges [49]). While some nodes are in regions with dense fiber-optic networks, many others are in the ocean or other sparsely connected regions.

$$\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{1}{ahc}\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{2(h/c + 1/a)}{h^2}\frac{\partial u(x,t)}{\partial t} = 0. \quad (11)$$

Previously, the telegraph equation has been widely used as a model for relativistic diffusion processes [58] and the probability densities of security prices evolving over time [59–62]. Here, in contrast, we find that it also models spatially distributed security prices evolving over time.

B. Effect of arbitrage on price propagation

As with the case of a single intermediate node examined earlier, we are again interested in how price fluctuations propagate, except now for a dense chain of trading centers. Substituting $u(x,t) \equiv e^{i(kx-\omega t)}$, where k is the wave number and ω is the angular frequency, we obtain the complex dispersion relation

$$h^2k^2 = \frac{1}{ac/h}\omega^2 + 2i\omega\left(\frac{1}{a} + \frac{1}{c/h}\right), \quad (12)$$

which we shall now analyze in the dense limit ($c/h \gg a$). For the two cases of $a, \omega \ll c/h$ and $a \ll c/h \ll \omega$, the dispersion relation simplifies to

$$k \approx \frac{1}{h}\sqrt{\frac{\omega}{a}}(1+i), \quad (13a)$$

$$k \approx \frac{\omega}{h\sqrt{ac/h}}\left(1 + i\frac{c/h}{\omega}\right), \quad (13b)$$

respectively. From Eqs. (13a) and (13b), we can derive the phase velocities ($\omega/\mathrm{Re}\,k$) and propagation lengths ($|\mathrm{Im}\,k|^{-1}$) in the three regimes bounded by the characteristic frequencies prescribed by Eq. (8), as summarized in Table I. (Since prices are nonconservative, the physical interpretation of the complex group velocity in this limit is not well established [63].)

In each frequency regime, we can see that the phase velocity is strongly subluminal, which is consistent with the telegraph equation disallowing superluminal propagation [52,53,56,58,64,65]. Note that in the absence of arbitrage couplings between neighboring trading centers (i.e., $a=0$), the phase velocity and propagation length both vanish, underlining the critical role of arbitrage at all frequencies.

Let us first examine the low-frequency regime ($\omega \ll a \ll c/h$). At low frequencies, the propagation length is much longer than the trading center spacing, h, and the phase velocity is much less than c, indicating that intermediate arbitrage has slowed—but not stopped—the propagation of price information through the chain, as parameterized by the large real refractive index,

$$\mathrm{Re}\,n \equiv \frac{c\,\mathrm{Re}\,k}{\omega} \approx \frac{c/h}{\sqrt{a\omega}} \gg 1. \quad (14)$$

In contrast, for the discrete two-center scenario we considered earlier, in which no arbitrage transactions were executed at intermediate points, we have $n=1$.

TABLE I. Propagation characteristics of price fluctuations within various frequency regimes along a dense chain of trading centers, as derived from dispersion relation [Eq. (12)]. Phase velocities, propagation lengths, and transparency are indicated.

Regime	Phase vel.	Prop. len.	Transparent
$\omega \ll a \ll c/h$	$c\frac{\sqrt{a\omega}}{c/h}$	$h\sqrt{\frac{a}{\omega}}$	Yes
$a \ll \omega \ll c/h$	$c\frac{\sqrt{a\omega}}{c/h}$	$h\sqrt{\frac{a}{\omega}}$	No
$a \ll c/h \ll \omega$	$c\sqrt{\frac{a}{c/h}}$	$h\sqrt{\frac{a}{c/h}}$	No

While the chain is transparent to low frequencies, the chain becomes opaque (i.e., the propagation length is much less than h) when we move to the intermediate-frequency regime ($a \ll \omega \ll c/h$) at which profitable arbitrage can take place, according to Eq. (8), and remains opaque in the high-frequency regime ($a \ll c/h \ll \omega$). The onset of opacity should have the effect of localizing price fluctuations and attenuating profitable arbitrage between non-nearest neighbor centers.

Such slowing or stopping of the propagation of pricing information due to arbitrage is somewhat analogous to the refraction and scattering of light by a dielectric medium, but novel in an econophysical context. We note that the effect exists independently of any communication latency intrinsic to the underlying hardware infrastructure, and would expect to observe a similar effect wherever tradable information enters a network "medium" capable of performing local arbitrage. This result also raises the possibility of establishing arbitrage analogs of other concepts from optics and acoustics, such as reflection and diffraction.

IV. CONCLUSIONS

In summary, we have demonstrated that light propagation delays present new opportunities for statistical arbitrage at the planetary scale, and have calculated a representative map of locations from which to coordinate such relativistic statistical arbitrage among the world's major securities exchanges. We furthermore have shown than for chains of trading centers along geodesics, the propagation of tradable information is effectively slowed or stopped by such arbitrage.

Historically, technologies for transportation and communication have resulted in the consolidation of financial markets. For example, in the nineteenth century, more than 200 stock exchanges were formed in the United States, but most were eliminated as the telegraph spread [66]. The growth of electronic markets has led to further consolidation in recent years [49]. Although there are advantages to centralization for many types of transactions, we have described a type of arbitrage that is just beginning to become relevant, and for which the trend is, surprisingly, in the direction of decentralization. In fact, our calculations suggest that this type of arbitrage may already be technologically feasible for the most distant pairs of exchanges, and may soon be feasible at the fastest relevant time scales for closer pairs.

Our results are both scientifically relevant because they identify an econophysical mechanism by which the propagation of tradable information can be slowed or stopped, and technologically significant, because they motivate the construction of relativistic statistical arbitrage trading nodes across the Earth's surface.

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